2014 Mathematics Institutes
The purpose of the 2014 Mathematics Institutes is to provide teachers with professional development focused on the instruction that supports process goals for students in mathematics. The emphasis of this professional development will be on fostering students' abilities to make mathematical connections and use effective and appropriate representations in mathematics.

Professional Development Instructions
A product of the 2014 Mathematics Institutes is a set of online professional development modules designed to be used by a group of teachers of a specific grade level or course, facilitated by a member of the team. Modifications could be made to adapt the professional development for more than one grade level/course or for large groups. Facilitators should review the activities and handouts in this guide prior to facilitating this professional development.

> **Virginia's Process Goals for Students**
>
> **SOL Vertical Articulation Technical Assistance Documents**
> - Grades K-3
> - Grades 3-6
> - Grades 5-8
> - Algebra
> - Geometry

Approximate Time	Facilitator Instructions	Materials
45 minutes total	**Module 1: Participant personal reflection on representations and connections** **Essential Question: What does it mean for students to represent and make connections in the mathematics classroom?**	- Five Types of Mathematical Representations - Process Goals for Students: Representations and Connections
	1) Think (individually)/Pair (share with a partner)/Share (with the whole group): – Pair: When you hear the term *representations*, what does it mean to you? Take turns sharing thoughts. – All: Look at the Five Types of Mathematical Representations document. Which of these was part of the previous discussion? – All: What does it mean for students to use effective and appropriate representations in the mathematics classroom? – Table: Why is it important for students to make connections in mathematics? Share an example of a connection students should make? – All: What does it mean for students to make connections in the mathematics classroom? 2) Whole group discussion: Compare and contrast the groups' discussion points with Process Goals for Students: Representations and Connections document. 3) Whole group discussion: What questions should be considered regarding representations and connections when planning for instruction? This list should be recorded for use in Module 5.	

| Approximate Time	Facilitator Instructions	Materials
45 minutes total	**Module 2: Mathematical task**	• Mathematical task (select the appropriate task)
	1) Select the appropriate mathematical task. Each person should solve the task in at least two different ways.	– Grade K-1, Grade 2
	2) Partner discussion: Identify representations and connections within the task and strategies for solving the task.	• Whole Number/ Number Sense Learning Progression
	– Table: What representations were used to solve the task? Explain your representation and strategy to others? What connections are evident in your solution?	• Van de Walle, J.A., Karp, K.S., Lovin, L.H. & Bay-Williams, J.M. (2014). *Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades K-2* (2nd ed.). (Vol. II). Pearson Education Inc.
	– Table: Capture at least 3 different ways the group solved the task and capture these on chart paper.	
	– All: Use a gallery walk to have members move among the posters, one person from the group stays with the poster to answer any questions and explains why representations were chosen. Others ask questions and look for connections among the representations they chose and others they see posted.	• Developing Number Sense by Building Number Relationships
	3) Whole group discussion: Identify the mathematical content addressed in the task.	
	– Where is this content found in the Whole Number/Number Sense Learning Progression document?	
	4) Brainstorm with a partner or small group: What are the early number relationships that move students from counting to reasoning?	
	5) Read and compare your list with the Developing Number Sense by Building Number Relationships document.	
	6) Reflect on the following quote: "To conceptualize a number as being made up of two or more parts is the most important relationship that can be developed about numbers." Van de Walle (2014, 108)	
	– Pair: Thinking about the mathematics that students learn as they go through elementary school and beyond, what connections or applications can you think of to this idea of thinking about a number in terms of parts? Jot down your thoughts.	

45 minutes total	**Module 3: Looking at student work**
	 Essential Question: How are students using representations and making connections in their work?
		• Student work
		   – [Grade K](#)
		   – [Grade 1](#)
		   – [Grade 2](#)
		 • [Student Work Recording Sheet](#)

	1) Select the appropriate student work corresponding to the mathematical task solved in Module 2. Choose the Kindergarten, Grade 1, or Grade 2 student work samples.
	 2) Partner or small group discussion: Use the Student Work Recording Sheet to note your observations:
	   – What representations are evident in each student's work? What are the similarities and differences?
	   – What does the student work tell us about their understanding? How do the representations give clues to student understanding?
	 3) Partner or small group discussion:
	   – Is there evidence that students are making connections within their own work?
	   – How could the student work be used to help all students make connections
	     o among representations,
	     o among strategies, or
	     o among mathematical ideas?
	 4) Whole group discussion:
	 How do the various student representations communicate
	   – student understanding of the mathematics;
	   – student generalizations of the mathematics; and
	   – mathematical relationships?	

40 minutes total	**Module 4: Planning for the use of representations and connections in instruction** **Essential Questions: What should be considered regarding representations and connections when planning for instruction? What representations can be used to help students develop strong part-part-whole ideas? What are the benefits and limitations of these representations?**	• [Representation Examples](#) • [Video of Kindergarteners](#): Visualizing Number Combinations, *The Teaching Channel* • Rekenrek directions (included on the last page of this guide) • [Video of Rekenrek number talk](#) ([transcript](#)) • [Part-Part-Whole Mat](#)
	Note: Additional extension activities from *Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades K-2* are referenced in the statewide facilitator [presentation](#). This resource was purchased for each division attending the 2014 Mathematics Institutes. The book activities are supplemental and are not required to complete this module.	

	1) Whole group: Consider some of the different representations that are used with young children for developing the ideas of part-part-whole. Review the Representation Examples document.
	   – What are the benefits and/or limitations for each of these representations? Record thoughts on the Representations-Benefits-Limitations document.
	2) Pair: How does your list of benefits and limitations compare to those listed in Mathematical Instructional Connections for Physical and Visual Representations document?
	3) Whole group: Read and discuss the Roles of the Teacher and Student in Making Connections and Using Representations document.
	   – Discuss what this looks like in your classrooms?
	4) Small group or whole group discussion: Consider the mathematical task from Module 2.
	   – Refer to the Whole Number/Number Sense Learning Progression document.
	      o What is the related content (prior knowledge) from previous and future grade(s)?
	      o How is the objective connected to the related content and to the real world?
	   – Refer to the Mathematics Instructional Connections for Physical and Visual Representations document.
	      o What representations/strategies may have been used to develop prior knowledge?
	      o What representations could be used to develop and reinforce understanding of the content?
	      o Which representations/strategies will model the mathematics and deepen and extend students' mathematical understanding?
	      o What are the strengths and limitations of the representation/strategy?
		• Counters
		• Representation-Benefits-Limitations
		• Roles of the Teacher and Student in Making Connections and Using Representations
		• Whole Number/ Number Sense Learning Progression
		• Mathematics Instructional Connections for Physical and Visual Representations

45 minutes total	**Module 5: Closure**	• Planning Mathematics Instruction: Essential Questions • Skip Fennell's NCTM President's Message • Student work – Grade 1 – Student 1-D
20 minutes	1) Small group discussion: Review and revise the list of planning questions brainstormed in Module 1: "What questions should be considered regarding representations and connections when planning for instruction?" 2) Whole group discussion: In what ways did your small group discussion support the following quote?	

	"Representation should be an important element of lesson planning. Teachers must ask themselves, 'What models or materials (representations) will help convey the mathematical focus of today's lesson?'" – Skip Fennell	
25 minutes	3) Small group discussion: Compare your list of questions to be considered regarding representations and connections when planning for instruction with the Planning Mathematics Instruction: Essential Questions document. – Highlight the questions you already think about when planning. – Which questions are new for you to think about? – Are the questions on the list you created earlier reflected on this document? 4) Whole group: Consider the student work for student 1-D. Assuming that the student's work is representative of your class, what would be your plan for instruction that will continue to move the students forward in their thinking? Consider the representations to be used in instruction and the connections students should make.	

Directions for Making a Rekenrek

Materials: 5 x 8 piece of cardstock
20 beads (10 red, 10 white or two contrasting colors)
Two pipe cleaners (any color)
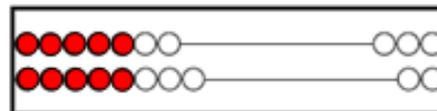
1) Cut 4 small slits in the cardboard (two on each of the shorter sides).
2) String 5 read and 5 white beads onto each of the two pipe cleaners.
3) Slip the ends of the pipe cleaners through the slits on the side of the cardboard so that the beads are on the front of the cardboard, and the ends of the pipe cleaners are on the back.