

**Institute Objective**

To improve mathematics instruction by providing district-level trainers with professional development resources focused on facilitating students' mathematical understanding through problem solving, communication, and reasoning.

Time	Notes	Materials
9:30 – 9:40AM  10 min	<p><b>Card Activity</b> (PowerPoint Slide 1)</p> <p><b>Group Arrangement:</b> Participants are seated at tables and asked to work in groups of 2 or 3.</p> <p><b>Objective:</b> To engage participants in problem solving, communication, and reasoning. This activity is also meant to highlight how it is often difficult to communicate one’s reasoning—even in a mathematics-neutral situation—and students need opportunities to practice both reasoning and its communication. This activity can also serve as a sample activity to engage 9–12 students in problem solving, communication, and reasoning without focusing on new content.</p> <ul style="list-style-type: none"> <li>• Distribute a deck of playing cards and a set of instructions to each group of participants.</li> <li>• Prior to physically conducting the experiment, ask the participants to formulate a hypothesis regarding its outcome and share this hypothesis with their group. (This part of the activity can be extended, depending on the level and progress of discussion).</li> <li>• Ask the participants to conduct the experiment to test their hypotheses and, if necessary, revise their thinking. If the participants struggle to identify the dynamics of the relationship between the two stacks of cards, suggest decreasing the number of cards in each pile or conducting the experiment with the cards face up.</li> <li>• To conclude this phase of the lesson, emphasize to the participants the importance of clear communication of reasoning and the difficulties it poses.</li> </ul>	<ul style="list-style-type: none"> <li>• One deck of playing cards for each group</li> <li>• One set of directions for each group. (<i>Card Task Handout</i>, printed on card stock and sliced horizontally.)</li> </ul>

Time	Notes	Materials
9:40 – 9:45AM  5 min	<p><b><u>Identifying Workshop Goals and Process Standards</u></b> (Slides 2–5)</p> <p><b>Group Arrangement:</b> Whole group</p> <p><b>Objective:</b> To identify the goals and framework of this professional development session:</p> <ul style="list-style-type: none"> <li>• The process goals for students found in the Virginia Standards of Learning</li> <li>• The meaning of teaching <u>through</u> problem solving</li> <li>• The rationale for focusing on problem solving, communication, and reasoning. (By focusing on a few processes, others will also be addressed.)</li> </ul>	<p>Handout showing the Virginia's Process Goals for Students (Process Standards) found in handouts from opening session</p>
9:45 – 10:15AM  30 min	<p><b><u>Exploring Functions</u></b> Task (Slide 6)</p> <p><b>Group Arrangement:</b> Groups of 2 or 3</p> <p><b>Objective:</b> The purpose of this activity is not to find the actual solution to each task, but to explore the approaches or strategies students might when working on it.</p> <ul style="list-style-type: none"> <li>• Direct the participants to work on each task in groups and share their brainstormed strategies with their table. (Think/Pair/Share)</li> <li>• Prompt the group as a whole to reflect upon and discuss these questions:               <ul style="list-style-type: none"> <li>○ What strategies could be used to solve each problem?</li> <li>○ What strategies would you expect your students to consider?</li> <li>○ What mathematical connections and representations did you consider while brainstorming these strategies? (Possible representations include Algebraic, Graphical, Numerical/Table, and Strategic Guessing and Checking.)</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>• Graphing calculators (at least 1 per group of 2 or 3 participants)</li> <li>• Graph paper (one sheet per participant)</li> <li>• Scrap paper (about one sheet per participant)</li> <li>• One copy of the handout <i>Exploring Functions</i> per participant</li> </ul>

Time	Notes	Materials
10:15 – 10:45AM 30 min	<p><b>Video: Exploring Functions</b> (Slide 7)</p> <p><b>Group Arrangement:</b> Whole group</p> <p><b>Objective:</b> To identify the process standards in practice through observation of a classroom video.</p> <ul style="list-style-type: none"> <li>• This video shows students engaged in solving the two exponential equations discussed by the participants during the previous activity. The video should be partitioned into 9 separate clips and paused between each clip to allow the participants to record instances in the video that embody the process standards. The time stamps for each clip are:               <ul style="list-style-type: none"> <li>○ Clip One: 1:01–2:57</li> <li>○ Clip Two: 2:57–4:24</li> <li>○ Clip Three: 4:24 – 5:13</li> <li>○ Clip Four: 5:13–6:13</li> <li>○ Clip Five: 6:13–6:47</li> <li>○ Clip Six: 8:10–8:49</li> <li>○ Clip Seven: 8:49–10:24</li> <li>○ Clip Eight: 10:24–11:35</li> <li>○ Clip Nine: 12:05–12:26</li> </ul> </li> </ul> <p>After all clips have been played, each participant should have a collection of observations for the process standards recorded.</p> <ul style="list-style-type: none"> <li>• Time permitting, ask the participants as a whole group to share the observations they recorded about each process standard.</li> </ul>	<ul style="list-style-type: none"> <li>• One copy of the handout <i>Process Standards Observations Tool</i> per participant.</li> <li>• Video: <i>Exploring Functions</i> <a href="http://www.learner.org/resources/series34.html?pop=yes&amp;pid=924">http://www.learner.org/resources/series34.html?pop=yes&amp;pid=924</a> <i>Exploring Functions</i>, from Teaching Math: A Video Library (5-8/9-12) used with permission by Annenberg Learner. <a href="http://www.learner.org">www.learner.org</a></li> </ul>
10:45 – 11:00AM 15 min	<p><b>BREAK</b></p>	

Time	Notes	Materials
11:00 – 11:10AM 10 min	<p><b>Examining Differences Between Tasks</b> (Slides 8–9)</p> <p><b>Group Arrangement:</b> Groups of 2 or 3, whole group</p> <p><b>Objective:</b> Participants identify the level of required thinking involved in three tasks focused on the same concept. This activity provides a foundation for introducing <i>cognitive demand</i> (Slide 9)</p> <ul style="list-style-type: none"> <li>• Distribute the Comparative Questions: Slope Tasks handout.</li> <li>• Direct the participants to examine the three tasks and answer the following questions in their groups (4 minutes).               <ul style="list-style-type: none"> <li>○ What do students need to know to solve each task?</li> <li>○ How are the tasks similar?</li> <li>○ How are the tasks different?</li> </ul> </li> <li>• Lead a brief whole-group discussion, soliciting several responses for each of the questions. (4 minutes)</li> <li>• Transition these responses to discussing the concept of cognitive demand (Slide 9) (2 minutes)               <ul style="list-style-type: none"> <li>○ Cognitive Demand (as defined in <i>Implementing Standards-Based Mathematics Instruction</i>): The kind and level of thinking required of students in order to successfully engage with and solve the task.</li> <li>○ The time spent discussing and analyzing the slope tasks will need to be limited. Participants will have more time to explore the cognitive demand of other mathematics tasks in the next activity.</li> </ul> </li> </ul>	<p>One copy of the handout  <i>Comparative Questions: Slope Tasks</i> per participant</p>

Time	Notes	Materials
<p>11:10 – 11:55PM 45 min</p>	<p><b><u>Task Sort Activity</u></b> (Slide 10)</p> <p><b>Group Arrangement:</b> Groups of 2 or 3</p> <p><b>Objective:</b> To sort a series of activities based on the level of cognitive demand for each of ten tasks. From this activity, participants will examine characteristics of high- and of low-level tasks.</p> <ul style="list-style-type: none"> <li>• Distribute the Task Sort handout to each participant, along with a group set of cut out tasks.</li> <li>• Direct the participant groups to (1) classify the cognitive demand of each task, (2) sort the tasks into two groups: high cognitive demand tasks and low cognitive demand tasks, and (3) record the task characteristics that prompted their sorting decisions. The results of this activity should be recorded in the Task Sort Organizer.</li> </ul>	<ul style="list-style-type: none"> <li>• One copy of the handout <i>Task Sort</i> per participant</li> <li>• One copy of the handout <i>Task Sort Organizer</i> per participant</li> <li>• One set of cut out cardstock tasks per group.</li> <li>• Scrap paper (about one sheet per participant)</li> <li>• Graphing calculators (at least 1 per group of 2 or 3 participants)</li> </ul>
<p>11:55AM – 12:00PM 5 min</p>	<p><b><u>Exit Slip: Recording Your Task Sort</u></b></p> <p><b>Group Arrangement:</b> Groups of 2 or 3</p> <p><b>Objective:</b> To collect data describing how each group classified the ten task-sort tasks.</p> <ul style="list-style-type: none"> <li>• Distribute one exit slip for each group. Instruct each group to record their consensus with respect to how the tasks should be sorted.</li> <li>• During lunch, compile the group data on the exit slips and enter the totals in the PowerPoint table (Slide 11).</li> </ul>	<ul style="list-style-type: none"> <li>• One copy of handout <i>Exit Slip</i> per group</li> <li>• One copy of handout <i>Task Sort Organizer</i> for the facilitator to tally the sorting results for each group.</li> </ul>
<p>12:00PM – 1:00 PM 1 hour</p>	<p><b><u>LUNCH</u></b></p>	

Time	Notes	Materials
<p>1:00 – 1:15PM 15 min</p>	<p><b><u>Task Sort Discussion</u></b> (Slide 11)</p> <p><b>Group Arrangement:</b> Whole group</p> <p><b>Objective:</b> To further examine and discuss specific tasks whose level of cognitive demand may be interpreted as either low or high. Factors creating this ambiguity may include students' prior knowledge, their exposure to similar types of problems, etc.</p> <ul style="list-style-type: none"> <li>• Display the table of task sort results generated by the exit slips.</li> <li>• Identify specific tasks that some groups classified as high cognitive demand but other groups classified as low cognitive demand.</li> <li>• Ask participants to volunteer their group's justification for sorting these tasks in that way. Allow other groups to respond to these justifications.</li> <li>• Transition this discussion to identifying other factors that may allow the cognitive demand to be interpreted in different ways.</li> </ul>	<p>Completed Task Sort Tally Chart</p>
<p>1:15 – 1:50PM 35 min</p>	<p><b><u>Characteristics of High &amp; Low Cognitive Demand Tasks</u></b> Slides (12–14)</p> <p><b>Group Arrangement:</b> Groups of 2 or 3, whole group</p> <p><b>Objective:</b> To identify the characteristics of high cognitive demand tasks and low cognitive demand tasks.</p> <ul style="list-style-type: none"> <li>• Distribute the chart paper and markers to each table.</li> <li>• Direct the groups to discuss and compile a list of characteristics of high cognitive demand tasks and characteristics of low cognitive demand tasks, recording both lists on chart paper.</li> <li>• In a whole-group discussion, ask each group to share one or two characteristics of high and low cognitive demand tasks. Write each of these contributions on chart paper, creating a whole-group list of characteristics.</li> <li>• Present the research-based identifying characteristics of high and low cognitive demand tasks (Slides 20–22). As time permits, discuss parallels between this list and the list generated by the group.</li> <li>• To conclude the discussion, present the characteristics of rich mathematical tasks</li> </ul>	<ul style="list-style-type: none"> <li>• Chart paper: one sheet per group and several sheets for the facilitator</li> <li>• Markers</li> </ul>

Time	Notes	Materials
<p>1:50 – 2:05PM 15 min</p>	<p>(Slide 23), one of which is high cognitive demand.</p> <p><b>Task Implementation</b> (Slides 15–22)</p> <p><b>Group Arrangement:</b> Individuals, whole group</p> <p><b>Objective:</b> To understand the Task Implementation Framework and identify factors maintain high-level cognitive demand during implementation and factors that lower the cognitive demand.</p> <ul style="list-style-type: none"> <li>• Present the idea that teachers’ instructional actions may affect the cognitive demand of the tasks used during that lesson. Refer to the Mathematics Task Framework on page 6 of <i>ISBMI</i> describing how tasks may change.</li> <li>• Prompt participants to brainstorm classroom factors that might lower the level of cognitive demand. As a whole group, ask participants to share some of these brainstorms. (2-minute share) Share the list of research-based factors (Slide 17).</li> <li>• Repeat the brainstorming and sharing activity, this time focusing on classroom factors that maintain the level of cognitive demand. (2-minute share) Share the list of research-based factors (Slide 18).</li> <li>• Present the discussion-oriented lesson structure and five habits for organizing and facilitating high-level classroom discussions. (For more information, see: Smith, M. S., Hughes, E. K., Engle, R. A., &amp; Stein, M. K. (2009). <i>Orchestrating discussions. Mathematics Teaching in the Middle School, 14</i>(9), 548–556.)</li> </ul>	<p>Stein, M. K., Smith, M. S., Henningsen, M. A., &amp; Silver, E. A. (2009). <i>Implementing standards-based mathematics instruction: A casebook for professional development</i> (2<sup>nd</sup> Ed.). New York, NY: Teachers College Press.</p>
<p>2:05 – 2:15PM 10 min</p>	<p><b>Triangle Task</b> (Slide 23)</p> <p><b>Group Arrangement:</b> Individual, Groups of 2 or 3</p> <p><b>Objective:</b> Become familiar with a mathematical task before analyzing student work on that task.</p> <ul style="list-style-type: none"> <li>• Distribute the Triangle Task to each participant.</li> <li>• Direct the participants to work individually on the task. (2 minutes)</li> <li>• Direct the participants to discuss the task in groups and identify the required elements for a satisfactory solution to the Triangle Task. Ask the groups to record their thinking about the task on chart paper.</li> </ul>	<ul style="list-style-type: none"> <li>• One copy of the handout <i>Triangle Task</i> per participant</li> <li>• Chart paper: one sheet per group</li> <li>• Markers</li> </ul>

Time	Notes	Materials
2:15 – 2:30PM 15 min	<b>BREAK</b>	
2:30 – 2:45PM 15 min	<p><b>Analyzing Student Work</b> (Slides 24–25)</p> <p><b>Group Arrangement:</b> Groups of 2 or 3</p> <p><b>Objective:</b> To practice selecting examples of student work to be used in a whole-class discussion, sequencing the examples to optimize the quality of the discussion and meet the mathematical goal of the lesson, and identifying connections found between the examples that might be discussed by students.</p> <ul style="list-style-type: none"> <li>• Distribute the student work on the triangle task and the rubric to the participants.</li> <li>• Direct the participants to work in groups to (1) select 4 to 5 examples of student work that would enable students to discuss the task and focus on the underlying mathematical ideas and (2) sequence the selected tasks in a way that will optimize the quality of the discussion.</li> <li>• Monitor group discussions. Prompt individual groups to reflect on different levels of student understanding and student misconceptions about the concepts being explored. Ask how the discussion might be structured to best help these students.</li> </ul>	<p>One copy of the handout <i>Triangle Task Student Solutions</i> per participant</p> <p>One copy of the handout <i>Triangle Task Rubric</i> per participant</p>
2:45 – 3:00PM 15 min	<p><b>Sequence of Sharing Student Work</b> (Slides 24–25)</p> <p><b>Group Arrangement:</b> Whole group</p> <p><b>Objective:</b> To think critically about different ideas for organizing a classroom discussion of a mathematical task.</p> <ul style="list-style-type: none"> <li>• Ask groups to share their selection and sequencing of tasks. Prompt these groups to identify their rationale for the selection and sequencing if it is not readily volunteered.</li> <li>• Prompt the whole group to identify mathematical connections between the examples of student work.</li> <li>•</li> </ul>	None

Time	Notes	Materials
3:00 – 3:15PM 15 min	<p><b><u>Next Steps: District Planning/Structure of Training</u></b> (Slide 26)</p> <p><b>Group Arrangement:</b> Groups of 2 or 3, whole group</p> <p><b>Objective:</b> To think critically about restructuring the activities and ideas presented during this workshop to suit the needs and purposes of individual districts.</p> <ul style="list-style-type: none"> <li>• Prompt the groups to discuss how today's activities might be adapted for professional development in their districts.</li> <li>• As a whole group, share some of the ideas brainstormed in the small-group discussion.</li> </ul>	None
3:15 – 3:30PM 15 min	<p><b><u>Questions, Facilitator Guide, and Look-fors</u></b></p> <p><b>Group Arrangement:</b> Whole group</p> <p><b>Objective:</b> To bring closure to the day's activities, present some additional resources, and answer participant questions.</p> <ul style="list-style-type: none"> <li>• Share the facilitator guide with the participants.</li> <li>• Share the process standards "Look-for" handout with the participants.</li> <li>• Field any questions the participants may have about these handouts or about the day's activities in general.</li> </ul>	<p>One copy of the <i>Facilitator Guide</i> per participant.</p> <p>One copy of the <i>Student Look-fors</i> handout per participant (distributed during opening session)</p>