

Introduction

The lessons in this section focus on comparing, adding, subtracting, multiplying, and dividing integers as well as describing the subsets of the real number system.

These lessons form an outline for your ARI classes, but you are expected to add other lessons as needed to address the concepts and provide practice of the skills introduced in the *ARI Curriculum Companion*.

Some of the lessons cross grade levels, as indicated by the SOL numbers shown below. This is one method to help students connect the content from grade to grade and to accelerate.

Standards of Learning

- 6.3 The student will
 - a) identify and represent integers;
 - b) order and compare integers; and
 - c) identify and describe absolute value of integers.
- 7.3 The student will
 - a) model addition, subtraction, multiplication, and division of integers; and
 - b) add, subtract, multiply, and divide integers.
- 8.2 The student will describe orally and in writing the relationships between the subsets of the real number system.

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* SOL 6.3 a, b

Lesson Summary

Students compare integers. (30 minutes)

Materials

Attached worksheets

Vocabulary

integer. The set of whole numbers, their opposites, and zero; $\{\dots-3, -2, -1, 0, 1, 2, 3, \dots\}$.

positive integer. An integer greater than zero.

negative integer. An integer less than zero. A negative integer is always less than a positive integer.

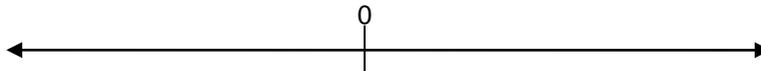
zero. An integer that is neither positive nor negative.

Warm-up

Distribute the “Comparing Integers Warm-up” worksheet, and have students complete it. Assist any students having difficulty. Discuss the answers.

Lesson

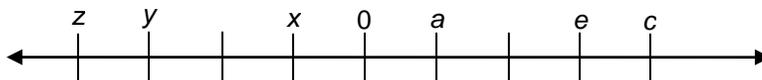
1. Draw 10 number lines on the board similar to the one shown below.



2. Ask students a series of 10 questions, such as the following:
 - John has \$3 in his wallet. Joan has \$5 but owes it to her mother. Who has more money? (John)
 - Monday’s high temperature was 7° . Tuesday’s was -3° . Which day had the higher temperature? (Monday)
 - Jamal is 10 years old. Maggie is 12. Who is older? (Maggie)
 - In Alaska, Friday’s temperature was -10° . On Saturday, it was -15° . Which day had the higher temperature? (Friday)

As you ask each question, write the corresponding integers on one of the number lines. Use a different number line for each question. Each time the students decide which number is greater, circle that number on its number line.

3. After asking the questions, point out to the students that the *larger* number on each number line is circled. Ask them to compare the number lines and say what these larger numbers have in common. (The larger number in each pair of numbers is always the number on the *right*.)
4. Erase the number lines, and put up the following number line:



Ask the students the following questions:

- Which letter, *a* or *e*, represents the larger number? (*e*)
- Which letter, *c* or *x*, represents the larger number? (*c*)
- Which letter, *z* or *y*, represents the larger number? (*y*)
- Which is larger, the number represented by *a* or *0*? (*a*)
- Which letter, *x* or *y*, represents the larger number? (*x*)

Ask the students to defend their answers.

5. Ask the students to work in pairs to make a rule about comparing integers. Students should decide that for numbers on the numbers line, the farther to the right they are the larger they are, and the farther to the left they are the smaller they are.
6. Distribute the “Comparing Integers Practice” worksheet, and allow students to work in pairs. Provide assistance as needed.

Reflection

Ask students to create their own number line with six letters representing numbers. Have them write five “Which is greater?” questions about this number line, exchange questions with a partner, and answer them.

Distribute the “Comparing Integers Assessment Questions” worksheet, and have the students complete it.

Name: _____

Comparing Integers Warm-up

1. Which of the following statements says that 100,100 is less than 101,001?
 - A $101,001 < 100,100$
 - B $100,100 = 101,001$
 - C $100,100 < 101,001$
 - D $100,100 > 101,001$

2. Which of the following statements should be read, “6,195,854 is greater than 6,195,845” ?
 - F $6,195,854 = 6,195,845$
 - G $6,195,854 > 6,195,845$
 - H $6,195,845 > 6,195,854$
 - J $6,195,854 < 6,195,845$

3. Which number would make the statement “ _____ $< 1,731,251$ ” true?
 - A 1,874,196
 - B 1,741,069
 - C 1,734,691
 - D 1,724,691

Name: **ANSWER KEY**

Comparing Integers Warm-up

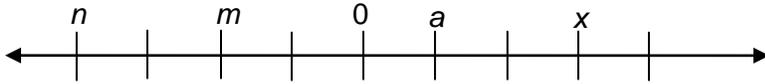
- Which of the following statements says that 100,100 is less than 101,001?
 - $101,001 < 100,100$
 - $100,100 = 101,001$
 - $100,100 < 101,001$
 - $100,100 > 101,001$

- Which of the following statements should be read, “6,195,854 is greater than 6,195,845” ?
 - $6,195,854 = 6,195,845$
 - $6,195,854 > 6,195,845$
 - $6,195,845 > 6,195,854$
 - $6,195,854 < 6,195,845$

- Which number would make the statement “ _____ $< 1,731,251$ ” true?
 - 1,874,196
 - 1,741,069
 - 1,734,691
 - 1,724,691

Name: _____

Comparing Integers Practice



True or False?

$a > m$ _____

$m < a$ _____

$n > a$ _____

$x > a$ _____

$m < x$ _____

$x < m$ _____

Fill in the symbol $>$ or $<$ to make each statement true. Use a number line to help you.

5 ___ -6

0 ___ -2

-4 ___ 4

18 ___ -16

14 ___ -7

-7 ___ -3

27 ___ -30

-20 ___ 14

-4 ___ -6

-8 ___ 0

-3 ___ -2

1 ___ -1

16 ___ 27

86 ___ -95

-37 ___ 22

-45 ___ -50

Write a sentence that explains how you can compare integers, using a number line.

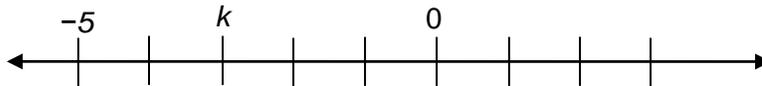
Name: _____

Comparing Integers Assessment Questions

1. On Monday, Bob received his bank statement. He had -5 dollars in his account. Hakeem had 5 dollars in his account. Which inequality below best represents the comparison of their accounts?

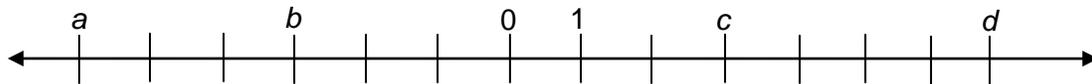
- A $-5 > 5$
- B $-5 = 5$
- C $-5 < 5$
- D $5 < -5$

2. On the number line below, k represents what number?



- E -8
- F -2
- G -3
- H 2

3. Which integers should replace letters a , b , c , and d on the number line below?



4. Fill in the symbol $>$ or $<$ to make each of the following statements about the number line above true:

a ____ b c ____ b a ____ d c ____ a

5. Order the following integers from smallest to largest: -5 , 8 , -11 , 15 , -30 .

6. True or false: $-13 > 11$ _____

Name: **ANSWER KEY**

Comparing Integers Assessment Questions

1. On Monday, Bob received his bank statement. He had -5 dollars in his account. Hakeem had 5 dollars in his account. Which inequality below best represents the comparison of their accounts?

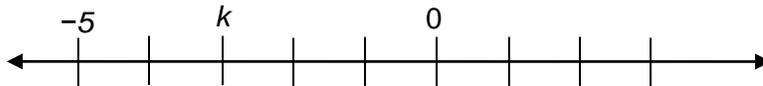
A $-5 > 5$

B $-5 = 5$

C $-5 < 5$

D $5 < -5$

2. On the number line below, k represents what number?



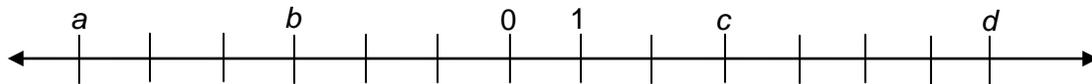
E -8

F -2

G -3

H 2

3. Which integers should replace letters a , b , c , and d on the number line below?



$a: -6, b: -3, c: 3, d: 7$

4. Fill in the symbol $>$ or $<$ to make each of the following statements about the number line above true:

a \leq b

c \geq b

a \leq d

c \geq a

5. Order the following integers from smallest to largest: $-5, 8, -11, 15, -30$.

$-30, -11, -5, 8, 15$

6. True or false: $-13 > 11$ False

* SOL 7.3

Lesson Summary

Students add integers, using Algeblocks™. (45 minutes)

Materials

Classroom set of Algeblocks™

Manual: *Algeblocks™ for Middle School*

Vocabulary

integer. The set of whole numbers, their opposites, and zero; {...-3, -2, -1, 0, 1, 2, 3,...}.

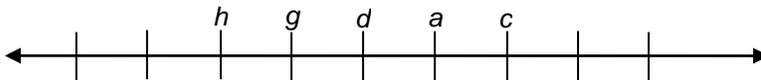
positive integer. An integer greater than zero.

negative integer. An integer less than zero. A negative integer is always less than a positive integer.

zero. An integer that is neither positive nor negative.

Warm-up

Write the following number line on the board.



Write the following statements on the board, and ask the students to insert < or > into each of them to make it true:

$$a _ d \quad (a > d)$$

$$h _ g \quad (h < g)$$

$$c _ d \quad (c > d)$$

$$a _ c \quad (a < c)$$

Ask students to defend their answers.

Lesson

In 2001, each middle school in Virginia received a classroom set of Algeblocks™ and a manual of lessons, *Algeblocks™ for Middle School*. In 2005, the Virginia Department of Education posted a streaming video, “Using Algeblocks™,” on its Web site. This online tool, designed to help classroom teachers learn how to use Algeblocks™, is available at http://www.vdoe.whro.org/A_Blocks05/index.html.

1. If your students are not familiar with Algeblocks™ prior to this lesson, have them work through the lessons in Unit 1 of the manual.
2. Once your students are acquainted with Algeblocks™, have them complete Unit 2 of the manual—the unit on adding integers.

Reflection

Have students explain in writing how to add integers.

* SOL 7.3

Lesson Summary

Students subtract integers, using Algeblocks™. (45 minutes)

Materials

Classroom set of Algeblocks™

Manual: *Algeblocks™ for Middle School*

Vocabulary

integer. The set of whole numbers, their opposites, and zero; {...-3, -2, -1, 0, 1, 2, 3,...}.

positive integer. An integer greater than zero.

negative integer. An integer less than zero. A negative integer is always less than a positive integer.

zero. An integer that is neither positive nor negative.

Warm-up

Write these problems on the board, and ask students to complete them:

$$\begin{array}{ll} -4 + 5 = x & (-4 + 5 = 1) \\ 6 + (-9) = x & [6 + (-9) = -3] \\ -3 + (-6) = x & [-3 + (-6) = -9] \end{array}$$

Lesson

In 2001, each middle school in Virginia received a classroom set of Algeblocks™ and a manual of lessons, *Algeblocks™ for Middle School*. In 2005, the Virginia Department of Education posted a streaming video, “Using Algeblocks™,” on its Web site. This online tool, designed to help classroom teachers learn how to use Algeblocks™, is available at http://www.vdoe.whro.org/A_Blocks05/index.html.

1. If your students are not familiar with Algeblocks™ prior to this lesson, have them work through the lessons in Unit 1 of the manual.
2. Once your students are acquainted with Algeblocks™, have them complete Unit 3 of the manual—the unit on subtracting integers.

Reflection

Have students explain in writing how to subtract integers.

* SOL 7.3

Lesson Summary

Students multiply and divide integers, using Algeblocks™. (Several class periods)

Materials

Classroom set of Algeblocks™

Manual: *Algeblocks™ for Middle School*

Vocabulary

integer. The set of whole numbers, their opposites, and zero; {...-3, -2, -1, 0, 1, 2, 3,...}.

positive integer. An integer greater than zero.

negative integer. An integer less than zero. A negative integer is always less than a positive integer.

zero. An integer that is neither positive nor negative.

Warm-up

Write these problems on the board, and ask students to complete them:

$$\begin{array}{ll} -4 - 5 = x & (-4 - 5 = -9) \\ 6 - (-9) = x & [6 - (-9) = 15] \\ -3 - (-6) = x & [-3 - (-6) = 3] \end{array}$$

Lesson

In 2001, each middle school in Virginia received a classroom set of Algeblocks™ and a manual of lessons, *Algeblocks™ for Middle School*. In 2005, the Virginia Department of Education posted a streaming video, “Using Algeblocks™,” on its Web site. This online tool, designed to help classroom teachers learn how to use Algeblocks™, is available at http://www.vdoe.whro.org/A_Blocks05/index.html.

1. If your students are not familiar with Algeblocks™ prior to this lesson, have them work through the lessons in Unit 1 of the manual.
2. Once your students are acquainted with Algeblocks™, have them complete Unit 4 of the manual—the unit on multiplying and dividing integers.

Reflection

Have students explain in writing how to multiply and divide integers.

* SOL 8.2

Lesson Summary

Students describe the relationships between the subsets of the real number system. (45 minutes)

Materials

Copies of the attached worksheets and templates
Scissors
Copies of the attached sample SOL questions

Vocabulary

natural number. The set of counting numbers {1, 2, 3, 4...}.

whole number. The set of all the natural numbers or counting numbers and zero {0, 1, 2, 3...}.

integer. The set of whole numbers and their opposites {...-2, -1, 0, 1, 2 ...}.

rational number. The set of all numbers that can be expressed as a fraction in the form $\frac{a}{b}$ where a and b are integers and b does not equal zero.

irrational number. The set of all nonrepeating, nonterminating decimals. An irrational number cannot be written in fraction form.

real number. The set of real numbers includes natural numbers, counting numbers, whole numbers, integers, rational and irrational numbers.

Warm-up

Write the following square roots on the board, and ask students to determine between which two whole numbers each square root lies.

$$\sqrt{51} \quad (7 \text{ and } 8)$$

$$\sqrt{95} \quad (9 \text{ and } 10)$$

$$\sqrt{14} \quad (3 \text{ and } 4)$$

$$\sqrt{34} \quad (5 \text{ and } 6)$$

Lesson

1. Arrange the students in pairs, and give each pair a copy of the “Real Numbers” template. Have the students cut apart the number cards.
2. Have the student pairs sort the number cards into any groupings that make sense to them. Circulate among the groups, and ask them to explain the reasoning behind their sorting.
3. Hold a class discussion on the attributes of the sets of numbers.
4. Hand out a copy of “Subsets of the Real Number System” to each group. Have the students cut out the five subsets and arrange them in any order.
5. Have the students sort the number cards onto the different subset cards. Discuss numbers that could belong to more than one subset. If students have forgotten the definitions of some of the subsets, this is a good time to reinforce them. Discuss the properties of each subset.
6. Pi (π), square root of 2 ($\sqrt{2}$), square root 5 ($\sqrt{5}$) and 1.232332333... should not have been used in step 5. Ask the students what is unusual about these numbers and why they do not fit into any of these subsets. Explain that any number that is nonrepeating and nonterminating is considered *irrational*. Define *nonrepeating* and *nonterminating*, and explain why irrational numbers cannot be written in fraction form ($\frac{a}{b}$).

7. Give a copy of the “Venn Diagram of the Subsets of the Real Number System” to each pair, and have the students write the names of the subsets in the appropriate places in the diagram. Then have them write the numbers in the appropriate subsets.
8. Have the students add one or two numbers to each subset on their diagram.

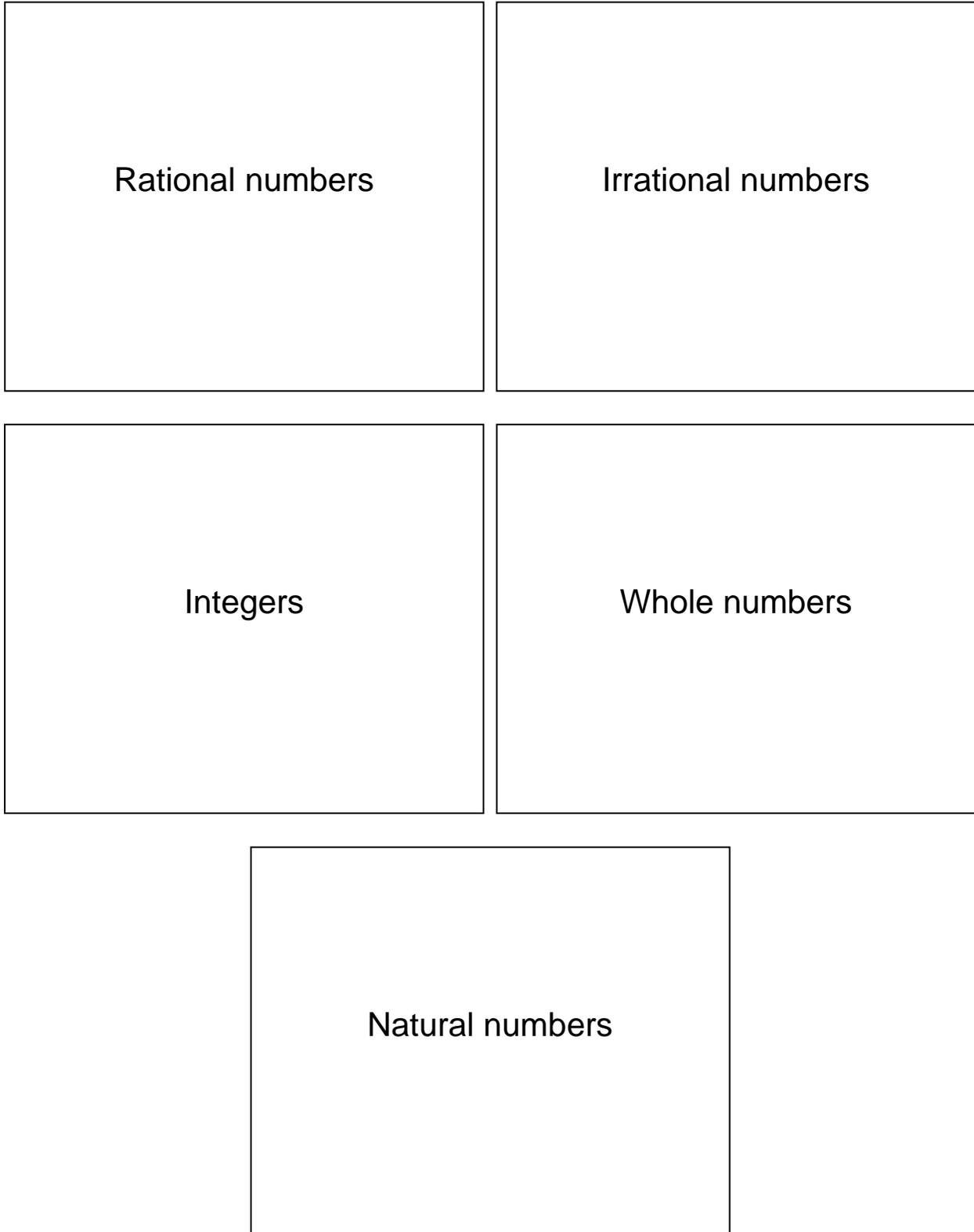
Reflection

Have each student create five number cards and trade them with a partner. Then have each student place the numbers on the Venn diagram in the correct place.

Real Numbers

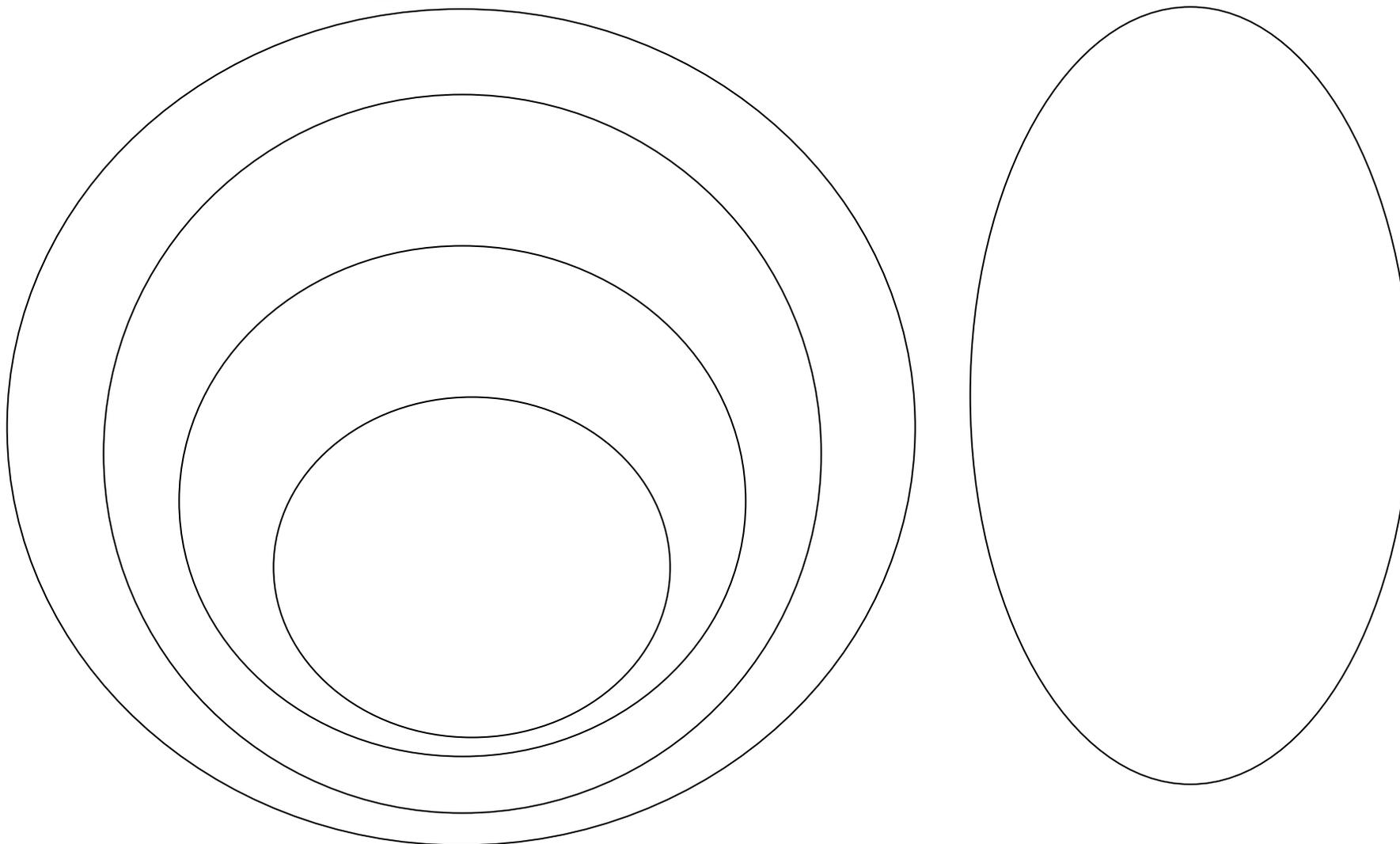
0	$\frac{1}{2}$	0.7	1
-3	$\sqrt{2}$	-0.9	π
-4.267	$-\frac{5}{17}$	14.8	-8
$\sqrt{36}$	$2.\overline{45}$	$\sqrt{5}$	1.232332333...

Subsets of the Real Number System



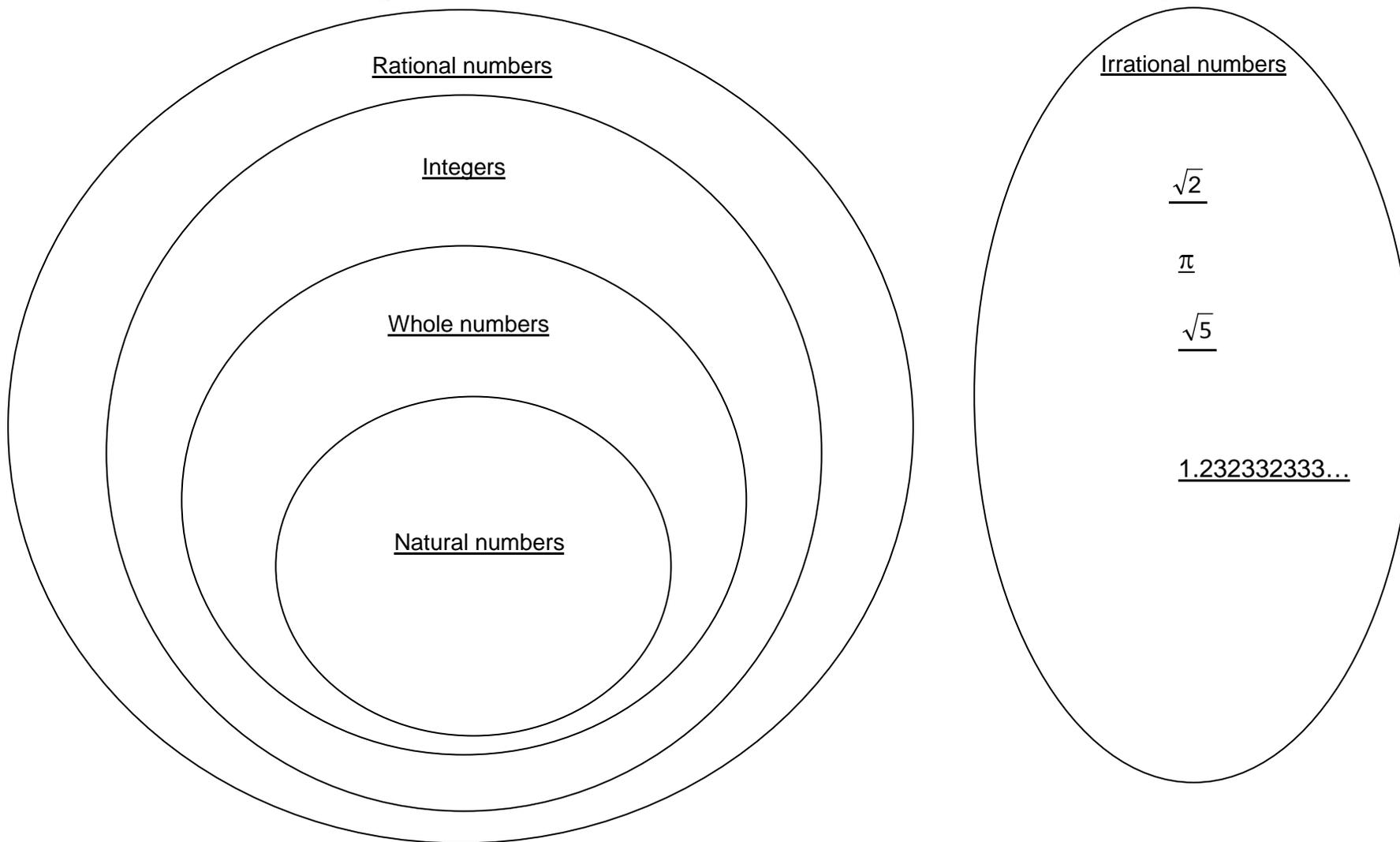
Name: _____

Venn Diagram of the Subsets of the Real Number System



Name: **ANSWER KEY**

Venn Diagram of the Subsets of the Real Number System

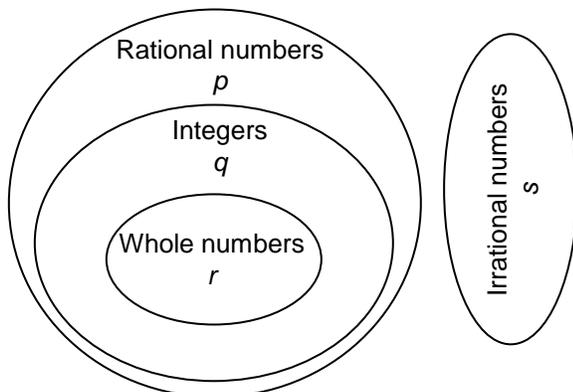


Name: _____

Sample SOL 8.2 Questions

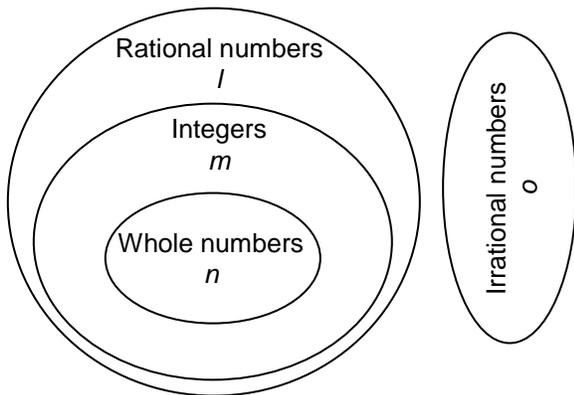
- Which statement is true?
A All integers are irrational numbers.
B All rational numbers are whole numbers.
C All irrational numbers are natural numbers.
D All integers are rational numbers.
- Which statement is true?
F All whole numbers are integers.
G All irrational numbers are integers.
H All natural numbers are irrational numbers.
J All rational numbers are natural numbers.

The diagram below shows the relationship among some of the subsets of the set of real numbers. The letters represent members of the sets.



- Christina wants to replace the letters with actual numbers. Which letter could be replaced with 0.5?
A p
B q
C r
D s

The diagram below shows the relationship among some of the subsets of the set of real numbers. The letters represent members of the sets.



4. Lenora wants to replace the letters with actual numbers. Which letter could be replaced with $\sqrt{2}$?
- F *l*
G *m*
H *n*
J *o*
5. Which *best* describes the set(s) of numbers to which $\sqrt{49}$ belongs?
- A {irrational}
B {rational, whole}
C {rational, whole, integers}
D {rational, integers}

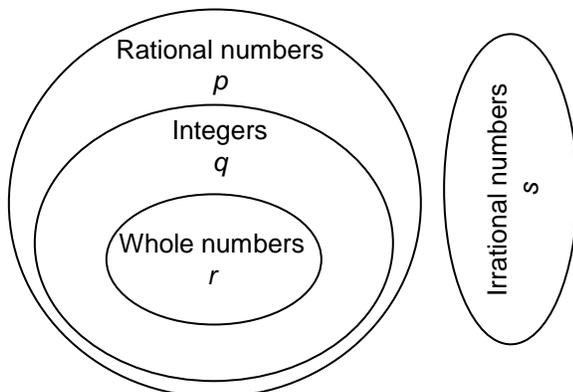
Name: **ANSWER KEY**

Sample SOL 8.2 Questions

- Which statement is true?
A All integers are irrational numbers.
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C All irrational numbers are natural numbers.
D All integers are rational numbers.

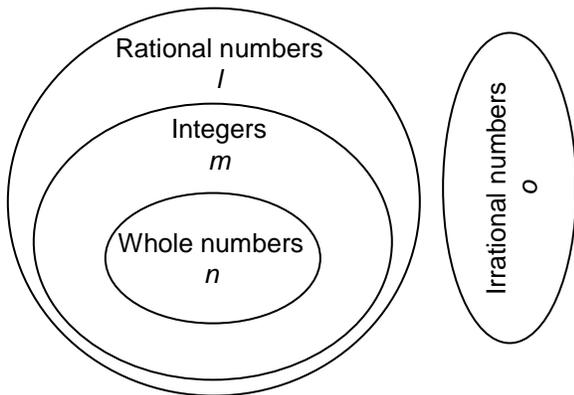
- Which statement is true?
F All whole numbers are integers.
G All irrational numbers are integers.
H All natural numbers are irrational numbers.
J All rational numbers are natural numbers.

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The diagram below shows the relationship among some of the subsets of the set of real numbers. The letters represent members of the sets.



4. Lenora wants to replace the letters with actual numbers. Which letter could be replaced with $\sqrt{2}$?
- | | |
|---|----------|
| F | <u>l</u> |
| G | m |
| H | n |
| J | o |
5. Which *best* describes the set(s) of numbers to which $\sqrt{49}$ belongs?
- | | |
|----------|------------------------------------|
| A | {irrational} |
| B | {rational, whole} |
| <u>C</u> | <u>{rational, whole, integers}</u> |
| D | {rational, integers} |